On the number of types in sparse graphs

Sebastian Siebertz

on joint work with Michał Pilipczuk and Szymon Toruńczyk

16.5.2019

I will present a proof sketch of our result showing that for every class of graphs $\mathcal{C}$ that is nowhere dense and for every first order formula $\phi(\bar{x}, \bar{y})$, whenever one draws a graph $G \in \mathcal{C}$ and a subset of its nodes A, the number of subsets of $A^{|\bar{y}|}$ which are of the form $\{\bar{v} \in A^{|\bar{y}|} : G \models \phi(\bar{u}, \bar{v})\}$ for some valuation $\bar{u}$ of $\bar{x}$ in G is bounded by $\mathcal{O}(|A|^{|\bar{x}|+\epsilon})$, for every $\epsilon > 0$. This provides optimal bounds on the vc-density of first-order definable set systems in nowhere dense graph classes.